

Entangling Mesoscopic Squeezed Vacuum States and Fock States in Dissipative Cavity QED System

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Abstract A scheme is proposed for generating the mesoscopic entanglement between the mesoscopic squeezed vacuum states and Fock states $\{|0\rangle, |2\rangle\}$ by considering both the two-photon interaction of N two-level atoms in cavities with high quality factor assisted by a strong driving field. Moreover, we derive the dissipative interaction models for two-photon interaction. The corresponding analytical expressions of the fidelities can be given. Our scheme can be realized in the current techniques on the cavity QED.

Keywords Multiparty entanglement · Cavity QED · Two-photon interaction

The squeezed vacuum state which is obtained from the squeezed operator functioned on the vacuum state have been investigated [1–12]. Indeed, Schrödinger discussed the nonexistence of quantum superpositions at the classical level in his famous “cat paradox” [1–3], and then stated that if the quantum superposition principles of the quantum dynamics are valid up to the macroscopic level, then the existence of quantum interference at the microscopic level necessary implies that the same phenomenon should occur between distinguishing macroscopic states. Therefore, whether there exists macroscopic quantum coherence becomes the core of debating quantum mechanics. Naturally, the decoherence process is also at the focus of the quantum measurement. Two types of evolution in quantum mechanics are introduced by Von Neumann in his collapse postulate [4]: the deterministic and unitary evolution associated with the Schrödinger equation, which establishes the correlation between states of microscopic system being measured and distinguishing classical states of the macroscopic measurement apparatus; and the probabilistic and irreversible process associated with measurement, which transforms the correlated state into a statistical mixture.

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In general, the transition between the microscopic and macroscopic worlds leads to the extensive studies of mesoscopic quantum states [5–12]. The coherent states and the squeezed states have extensively been investigated [5–12], and then the corresponding superpositions and entangled states have been prepared in cavity QED [5–10] and in trapped ions system [11, 12]. For example, a scheme [13] for squeezed vacuum measurements without homodyning has been proposed by Wenger *et al.*, following the theoretical proposal presented by Fiurášek and Cerf [14]. The scheme can be utilized to measure the squeezing and purity of single mode squeezed vacuum state, providing a powerful tool for the study on the squeezed vacuum state. Recently, Chen *et al.* have proposed a novel scheme for the generation of superposition and entanglement of mesoscopic squeezed vacuum states in cavity QED by considering the two-photon interaction of N two-level atoms in a cavity with high quality factor, assisted by a strong driving field [15–17]. In this scheme, the macroscopic squeezed vacuum “Schrödinger cats” and a number of multiparty mesoscopic entangled states can be prepared by virtue of the detuned cavity and the applied coherent field. However, only a little attention has been paid on the study of the entanglement between the squeezed vacuum state and Fock states and the influence from dissipation on the preparation of them.

In this letter, a scheme is proposed for entangling the mesoscopic squeezed vacuum states and the microscopic Fock states by the two-photon interaction of N two-level atoms in cavities with high quality factor assisted by strong driving fields. Moreover, in the case that the cavity decay rates are considered, the corresponding analytical expressions of the fidelity can be given, based on the dissipative two-photon interaction Hamiltonian. Our scheme can be realized in the current techniques on the cavity QED.

The Jaynes-Cummings model [18] based on the one-photon interaction can describe the interaction of a two-level atom with a single mode of the electromagnetic field, which is the simplest and most fundamental quantum model where the cavity mode decay has not been considered. Here, we first consider a spatially narrow bunch of N identical two-level atoms in a dissipative two-photon interaction with M modes in a cavity of high quality factor, driven additionally by an external classical field. The Hamiltonian under the cavity mode decay can be expressed as (assuming $\hbar = 1$)

$$\begin{aligned}
 H_s = & \omega_0 \sum_{j=1}^N S_{z,j} + \sum_{i=1}^M \omega_{ci} a_i^\dagger a_i + \sum_{j=1}^N \sum_{i=1}^M g_{ij} (a_i^{+2} S_j^- + a_i^2 S_j^+) \\
 & + \Omega \sum_{j=1}^N (e^{-i\omega_L t} S_j^+ + e^{i\omega_L t} S_j^-) - i \frac{\kappa}{2} \sum_{i=1}^M a_i^\dagger a_i,
 \end{aligned} \tag{1}$$

where ω_0 , ω_{ci} , and ω_L are the frequencies of the resonant transition between $|e\rangle$ and $|g\rangle$, the cavity modes, and the classical laser field, respectively. κ denotes the decay rate of the cavity modes. $S_{z,j} = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$, $S_j^+ = |e\rangle\langle g|$, and $S_j^- = |g\rangle\langle e|$, a_i^\dagger and a_i are the creation and annihilation operators for the cavity mode, respectively, and g and Ω are the coupling constants of each atom to the cavity modes and to the driving field, respectively. In the rotating frame with respect to the driving field frequency ω_L , the Hamiltonian is given by

$$\begin{aligned}
 H_r = & \sum_{j=1}^N \Delta S_{z,j} + \sum_{i=1}^M \delta_i a_i^\dagger a_i + \Omega \sum_{j=1}^N (S_j^+ + S_j^-) \\
 & + \sum_{j=1}^N \sum_{i=1}^M g_i (a_i^{+2} S_j^- + a_i^2 S_j^+) - i \frac{\kappa}{2} \sum_{i=1}^M a_i^\dagger a_i,
 \end{aligned} \tag{2}$$

where $\Delta = \omega_0 - \omega_L$ and $\delta_i = \omega_{ci} - \omega_L/2$. Assume that $\Delta = 0$ is satisfied, in the interaction picture we have

$$\begin{aligned}
 H_i &= e^{iH_{r0}t} H_{ri} e^{-iH_{r0}t} \\
 &= \frac{1}{2} \sum_{j=1}^N \sum_{i=1}^M g_{ij} [|+\rangle_{jj} \langle +| - |-\rangle_{jj} \langle -| + e^{i2\Omega t} |+\rangle_{jj} \langle -| \\
 &\quad - e^{-i2\Omega t} |-\rangle_{jj} \langle +|] a_i^2 e^{-i2\delta_i t} e^{-\kappa t} \\
 &\quad + \frac{1}{2} \sum_{j=1}^N \sum_{i=1}^M g_{ij} [|+\rangle_{jj} \langle +| - |-\rangle_{jj} \langle -| + e^{-i2\Omega t} |-\rangle_{jj} \langle +| \\
 &\quad - e^{i2\Omega t} |+\rangle_{jj} \langle -|] a_i^{+2} e^{i2\delta_i t} e^{\kappa t}, \tag{3}
 \end{aligned}$$

where

$$H_{r0} = \sum_{i=1}^M \delta_i a_i^+ a_i + \Omega \sum_{j=1}^N (S_j^+ + S_j^-) - i \frac{\kappa}{2} \sum_{i=1}^M a_i^+ a_i, \tag{4}$$

and

$$H_{ri} = \sum_{j=1}^N \sum_{i=1}^M g_{ij} (a_i^{+2} S_j^- + a_i^2 S_j^+), \tag{5}$$

and where $|\pm\rangle_j = (|g\rangle_j \pm |e\rangle_j)/\sqrt{2}$ and $S_{jx} |\pm\rangle_j = (S_j^+ + S_j^-) |\pm\rangle_j = \pm |\pm\rangle_j$. In the strong driving regime $\Omega \gg \delta_i, g_{ij}, \kappa$, a rotating-wave approximation can be utilized and then the effective Hamiltonian is obtained as follows

$$\begin{aligned}
 H_{ieff} &= \frac{1}{2} \sum_{j=1}^N \sum_{i=1}^M g_{ij} (|+\rangle_{jj} \langle +| - |-\rangle_{jj} \langle -|) (a_i^2 e^{-i2\delta_i t} e^{-\kappa t} + a_i^{+2} e^{i2\delta_i t} e^{\kappa t}) \\
 &= \frac{1}{2} \sum_{j=1}^N \sum_{i=1}^M S_{jx} g_{ij} (a_i^2 e^{-i2\delta_i t} e^{-\kappa t} + a_i^{+2} e^{i2\delta_i t} e^{\kappa t}). \tag{6}
 \end{aligned}$$

In our model, when $\delta = 2\Omega$ and $|\delta| \gg g_i$, (3) will turn to the effective Hamiltonian of dissipative two-photon JC interaction in the dressed basis $|\pm\rangle_j$,

$$H_{TJC}^+ = \frac{1}{2} \sum_{j=1}^N \sum_{i=1}^M g_{ij} [|+\rangle_{jj} \langle -| a_i^2 e^{-i2\delta_i t} e^{-\kappa t} + |-\rangle_{jj} \langle +| a_i^{+2} e^{i2\delta_i t} e^{\kappa t}]. \tag{7}$$

In the following, we will use (6) to generate the entangled state between the two mesoscopic squeezed vacuum states under the considering of the cavity mode decay. We consider the case that $N = 1, M = 1$, with two single-mode cavities, and the atom-field is initially in $|g\rangle |0\rangle_1 |0\rangle_2$. After the interaction times t_1 and t_2 of the atom with cavities 1 and 2, respectively, the total state evolves into

$$\begin{aligned}
 |\psi\rangle_{1decay} &= e^{-iH_{ieff}t} |g\rangle |0\rangle_1 |0\rangle_2 \\
 &= e^{-iH_{ieff}t} \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle) |0\rangle_1 |0\rangle_2
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{\sqrt{2}} [|+\rangle S_g[\xi_1(t_1), \xi_1(-t_1)] S_g[\xi_2(t_2), \xi_2(-t_2)] \\
 &\quad + |-\rangle S_g[-\xi_1(t_1), -\xi_1(-t_1)] S_g[-\xi_2(t_2), -\xi_2(-t_2)] |0\rangle_1 |0\rangle_2 \\
 &= \frac{1}{\sqrt{2}} (|+\rangle |\xi_1\rangle_g |\xi_2\rangle_g + |-\rangle |-\xi_1\rangle_g |-\xi_2\rangle_g), \tag{8}
 \end{aligned}$$

where the generalized squeezed operator is defined by $S_g[\xi_l(t), \xi_l(-t)] = e^{\frac{1}{2}[\xi_l(t)a_l^2 + \xi_l(-t)a_l^{+2}]}$, different from the standard squeezed operator $S_s[\xi'_l(t)] = e^{\frac{1}{2}[\xi'_l(t)a_l^2 - \xi_l'^*(t)a_l^{+2}]}$, $\xi_l(t) = -\int_0^t i g_l e^{-i2\delta_i t'} e^{-\kappa t'} dt' = g_l [2\delta_i + i\kappa] [e^{-\kappa t} e^{-i2\delta_i t} - 1] / [4\delta_i^2 + \kappa^2]$ ($l = 1, 2$) and $|\xi_l\rangle_g$ is the generalized squeezed vacuum states of the cavity modes, which is different from $t|\xi'_l\rangle_s = S_s[\xi'_l(t)]|0\rangle$. Equation (8) describes a generalized three-party entangled state between one microscopic and two mesoscopic systems. If the cavity mode decay rate $\kappa = 0$, (8) will return to the ideal case $|\psi\rangle_{ideal}$, which is expressed as

$$|\psi\rangle_{ideal} = \frac{1}{\sqrt{2}} (|+\rangle |\xi'_1\rangle_s |\xi'_2\rangle_s + |-\rangle |-\xi'_1\rangle_s |-\xi'_2\rangle_s). \tag{9}$$

Therefore, we can give the fidelity of generation of the three-party entangled state in (8):

$$\begin{aligned}
 F_1 &= |_{ideal}\langle\psi||\psi\rangle_{decay}|^2 \\
 &= \frac{1}{4} \{ |_s\langle\xi'_1||\xi_1\rangle_{gs} \langle\xi'_2||\xi_2\rangle_g|^2 + |_s\langle-\xi'_1||-\xi_1\rangle_{gs} \langle-\xi'_2||-\xi_2\rangle_g|^2 \\
 &\quad + 2 \langle\xi'_1||\xi_1\rangle_{gs} \langle\xi'_2||\xi_2\rangle_{gs} \langle-\xi'_1||-\xi_1\rangle_{gs} \langle-\xi'_2||-\xi_2\rangle_g \} \\
 &= |_s\langle\xi'_1||\xi_1\rangle_g|^2 |_s\langle\xi'_2||\xi_2\rangle_g|^2 \\
 &= \cosh |\xi'_1| \cosh |\xi'_2| \cosh |\xi_1| \cosh |\xi_2| \\
 &\quad \times \left| \sum_{m,n,k,l}^{\infty} \frac{e^{i(m+n+k+l)(\theta_1+\theta_2+\theta'_1+\theta'_2)} (\tanh |\xi'_1| \tanh |\xi'_2| \tanh |\xi_1| \tanh |\xi_2|)^{(m+n+k+l)}}{2^{2(m+n+k+l)} (2m)! (2n)! (2k)! (2l)!} \right|, \tag{10}
 \end{aligned}$$

where

$$\begin{aligned}
 \xi'_i(t) &= g_i (e^{-i2\delta_i t} - 1) / (4\delta_i^2 + \kappa^2) \quad (i = 1, 2), \\
 \xi_i(t) &= g_i (2\delta_i + i\kappa) (e^{-\kappa t} e^{-i2\delta_i t} - 1) / (4\delta_i^2 + \kappa^2) \quad (i = 1, 2), \\
 |\xi'_i(t)| &= \left| \frac{g_i \sin(\delta_i t)}{\delta_i} \right|, \\
 |\xi_i(t)| &= \frac{g_i}{4\delta_i^2 + \kappa^2} \{ [2\delta_i e^{-\kappa t} \sin 2\delta_i t - \kappa (e^{-\kappa t} \cos 2\delta_i t - 1)]^2 \\
 &\quad + [2\delta_i (e^{-\kappa t} \cos 2\delta_i t - 1) - \kappa e^{-\kappa t} \sin 2\delta_i t]^2 \}^{\frac{1}{2}}, \\
 \theta'_i(t) &= -\arctan(\tan^{-1} \delta_i t), \\
 \theta_i(t) &= \arctan \frac{2\delta_i (e^{-\kappa t} \cos 2\delta_i t - 1) - \kappa e^{-\kappa t} \sin 2\delta_i t}{2\delta_i e^{-\kappa t} \sin 2\delta_i t - \kappa (e^{-\kappa t} \cos 2\delta_i t - 1)}. \tag{11}
 \end{aligned}$$

If $\xi'_1 = \xi'_2$ and $\xi_1 = \xi_2$, through complex derivation, we can obtain

$$F_1 = (\cosh |\xi'_1| \cosh |\xi_1|)^2 \times \left| \sum_{m,n} \frac{e^{i(m+n)(\theta_1+\theta'_1)} (\tanh |\xi'_1| \tanh |\xi_1|)^{(m+n)}}{2^{2(m+n+k+l)} (2m)! (2n)!} \right|^2. \tag{12}$$

According to [15–17], (8) can also be written as

$$|\psi\rangle = \frac{1}{2} [|g\rangle (|\xi_1\rangle_g |\xi_2\rangle_g + |-\xi_1\rangle_g |-\xi_2\rangle_g) + |e\rangle (|\xi_1\rangle_g |\xi_2\rangle_g - |-\xi_1\rangle_g |-\xi_2\rangle_g)]. \tag{13}$$

Then the atom is detected in the basis $\{|g\rangle, |e\rangle\}$, we can obtain the macroscopic entangled squeezed vacuum states

$$|\psi\rangle_{\pm} = \frac{1}{\sqrt{2}} [|\xi_1\rangle_g |\xi_2\rangle_g \pm |-\xi_1\rangle_g |-\xi_2\rangle_g], \tag{14}$$

respectively. If the atom pass continuously through the n same cavities as above before the measurement, we can acquire bigger macroscopic entangled squeezed states after the detection of the atom

$$|\psi\rangle_{\pm} = \frac{1}{\sqrt{2^{n/2}}} \left[\prod_{j=1}^n |\xi_j\rangle_g \pm (-1)^{n/2} \prod_{j=1}^n |-\xi_j\rangle_g \right]. \tag{15}$$

It is noted that this is a more macroscopic entangled squeezed state involving in n mesoscopic systems, called as generalized macroscopic GHZ state in which the cavity mode decay has been considered. The corresponding fidelity is $F_1^{n/2}$ when $\xi'_j = \xi'_1$ and $\xi_j = \xi_1, j = 1, \dots, n$.

We now turn to generate the entangled states between the generalized squeezed vacuum states and Fock states. Supposing that one atom passes sequentially through cavity 1 for interaction time t_1 and cavity 2 for interaction time t_2 , the interactions between the atom and the cavity 1 and the cavity 2 are controlled by (7) and (6) respectively, with $M = 1$ for cavities 1 and 2. If the system is initially in $|g\rangle |0\rangle_1 |0\rangle_2$, after the interaction time $t_1 + t_2$, we can obtain in the interaction picture

$$\begin{aligned} |\psi\rangle_{2decay} &= e^{-iH_{ieff}t_2} e^{-iH'_{TJC}t_1} |g\rangle |0\rangle_1 |0\rangle_2 \\ &= e^{-iH_{ieff}t_2} e^{-iH'_{ieff}t_1} \frac{1}{\sqrt{2}} (|+\rangle_1 S_g[\xi_2(t_2), \xi_2(-t_2)] e^{-iH'_{TJC}t_1} \\ &\quad + |-\rangle S_g[-\xi_2(t_2), -\xi_2(-t_2)] e^{-iH'_{TJC}t_1}) |0\rangle_1 |0\rangle_2 \\ &= \frac{1}{\sqrt{2}} \left\{ \left[\cos\left(\frac{\sqrt{2}g}{\kappa} \sinh \frac{\kappa t_1}{2}\right) |+\rangle_1 |0\rangle_1 \right. \right. \\ &\quad \left. \left. - i e^{i\kappa t_1/2} \sin\left(\frac{\sqrt{2}g}{\kappa} \sinh \frac{\kappa t_1}{2}\right) |-\rangle_1 |2\rangle_1 \right] |\xi_2(t_2)\rangle_g + |-\rangle_1 |0\rangle_1 |-\xi_2(t_2)\rangle_g \right\}. \end{aligned} \tag{16}$$

When $\sinh \frac{\kappa t_1}{2} = \frac{\kappa\pi}{2\sqrt{2}g}$, we have

$$|\psi\rangle_{2decay} = \frac{1}{\sqrt{2}} \{ -i e^{i\kappa t_1/2} |2\rangle_1 |\xi_2(t_2)\rangle_g + |0\rangle_1 |-\xi_2(t_2)\rangle_g \} |-\rangle_1. \tag{17}$$

If the cavity decay rates are zero, (16) and (17) will reduce to the ideal case [15–17], respectively,

$$\begin{aligned}
 |\psi\rangle_{2ideal} &= e^{-iH_{ieff}t_2} e^{-iH'_{TJC}t_1} |g\rangle |0\rangle_1 |0\rangle_2 \\
 &= e^{-iH_{ieff}t_2} e^{-iH'_{TJC}t_1} \frac{1}{\sqrt{2}} (|+\rangle_1 S_s[\xi_2(t_2), \xi_2(-t_2)] e^{-iH'_{TJC}t_1} \\
 &\quad + |-\rangle_1 S_s[-\xi_2(t_2), -\xi_2(-t_2)] e^{-iH'_{TJC}t_1} |0\rangle_1 |0\rangle_2 \\
 &= \frac{1}{\sqrt{2}} \{ [\cos(\sqrt{2}gt_1) |+\rangle_1 |0\rangle_1 - i \sin(\sqrt{2}gt_1) |-\rangle_1 |2\rangle_1] |\xi_2(t_2)\rangle_s \\
 &\quad + |-\rangle_1 |0\rangle_1 |-\xi_2(t_2)\rangle_s \}, \tag{18}
 \end{aligned}$$

and

$$|\psi\rangle_{2ideal} = \frac{1}{\sqrt{2}} \{ -i |2\rangle_1 |\xi'_2(t_2)\rangle_s + |0\rangle_1 |-\xi'_2(t_2)\rangle_s \} |-\rangle_1, \tag{19}$$

where $\sqrt{2}gt_1 = \frac{\pi}{2}$. Therefore, we can generate deterministically the entanglement between the generalized squeezed states or the standard squeezed states and Fock states.

Through complex computation, its fidelity can be obtained

$$\begin{aligned}
 F_2 &= |_{2ideal} \langle \psi | | \psi \rangle_{2decay} |^2 \\
 &= \frac{1}{2} (1 - e^{\kappa t_1/2})^2 |_{s} \langle \xi'_2 | | \xi_2 \rangle_g |^2 \\
 &= \frac{1}{2} (1 - e^{\kappa t_1/2})^2 \times (\cosh |\xi'_2| \cosh |\xi_2|) \\
 &\quad \times \left| \sum_{m,n} \frac{e^{i(m+n)(\theta_1+\theta'_1)} (\tanh |\xi'_2| \tanh |\xi_2|)^{(m+n)}}{2^{2(m+n)} (2m)! (2n)!} \right|, \tag{20}
 \end{aligned}$$

where

$$\begin{aligned}
 |\xi'_2(t)| &= \left| \frac{g_2 \sin(\delta_2 t)}{\delta_2} \right|, \\
 |\xi_2(t)| &= \frac{g_2}{4\delta_2^2 + \kappa^2} \{ [2\delta_2 e^{-\kappa t} \sin 2\delta_2 t - \kappa (e^{-\kappa t} \cos 2\delta_2 t - 1)]^2 \\
 &\quad + [2\delta_2 (e^{-\kappa t} \cos 2\delta_2 t - 1) - \kappa e^{-\kappa t} \sin 2\delta_2 t]^2 \}^{\frac{1}{2}}, \tag{21} \\
 \theta'_1(t) &= -\arctan \left(\tan^{-1} \frac{\delta_1 t}{2} \right), \\
 \theta_1(t) &= \arctan \frac{2\delta_1 e^{-\kappa t/2} \sin \delta_1 t - \kappa (e^{-\kappa t/2} \cos \delta_1 t - 1)}{2\delta_1 (e^{-\kappa t/2} \cos \delta_1 t - 1) + \kappa e^{-\kappa t/2} \sin \delta_1 t}.
 \end{aligned}$$

Our proposal is experimentally feasible with current available cavity QED techniques. Both microwave and optical regimes may be utilized for implementation of our scheme in the cases of both open and closed cavities [19–27]. For example, in the microwave regime, we assume that $\delta/2\pi = 1$ MHz, $\Omega/2\pi = 1$ GHz, and $g_{ij}/2\pi = 0.050$ MHz [25]. So the rotating-wave approximation in (6) is satisfied. The lifetime of the circular Rydberg atom

state with principle quantum 51 is about 30 ms, much longer than the cavity decay time T_{cav} (0.85 ms) in the case of multiphoton inside [29]. If we assume that the cavity size is $L = 27.5$ mm and the atomic velocity is 500 m/s [25], the interaction time of the atoms with the cavity mode is $T_i = 5.5 \times 10^{-2}$ ms, which is much shorter than T_{cav} , and then $T_{cav}/T_i = 15.45$, meaning that if the detection time T_d is equal to the interaction time T_i , and if the distance between neighbor cavities is half of the cavity size, we can sequentially manipulate about *five* cavities involved by detection on a single atom to realize the first part of our scheme. The second part can be implemented by setting $\delta = 2\Omega$ for the first cavity and $\delta/2\pi = 1$ MHz, $\Omega/2\pi = 1$ GHz, and $g_{ij}/2\pi = 0.05$ MHz for the second cavity.

In conclusion, we have been proposed a feasible scheme for realizing the entanglement between the mesoscopic squeezed vacuum states and Fock states $\{|0\rangle, |2\rangle\}$. In our scheme, we have utilized the interactions between single atom and many cavity modes in many cavities of dissipations, which are based on the dissipative two-photon Hamiltonians assisted by strongly-assisted driving classical field. We can also give fidelities for generation of the entangled states between the generalized squeezed vacuum states and Fock states. All obtained results are suitable for realization in the microscopic and optical regimes in cavity QED experiments, with atoms flying through the cavities or conveniently trapped inside them [8]. We argue that our schemes can be realized by current techniques in cavity QED [28, 29].

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